“RMS Power”

Discussion in the rec.radio.amateur.homebrew newsgroup showed a widespread misunderstanding of the meaning and importance of RMS and average values of voltage, current, and power. So I’ve put together this explanation which uses as little math as possible, and hopefully explains the concepts in an intuitive way.

The Meaning of Average

Before I can proceed, it’s important to explain just what average means. Fortunately, its meaning when dealing with waveforms is essentially the same as its common meaning. If we were to sample a waveform (that is, a graph of voltage, current, power, etc. versus time) at equally spaced times, then add up their values and divide by the number of samples, we’d have approximately the average value of that voltage, current, power, or whatever the waveform represents. The smaller the time intervals between samples, the more accurate the average will be. The mathematical operation of integration is a way to find what the value would be if we could shrink the time interval extremely close to zero, and it’s needed if we want to calculate the exact average value of some waveforms. But I’m going to use square waves for this explanation, and we can easily see their average values without any math at all.

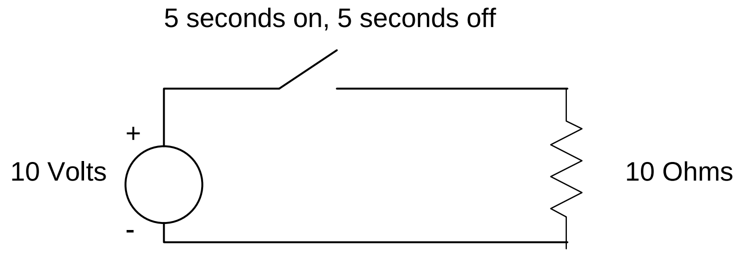
A periodic waveform is one that repeats in an identical fashion, over and over. The period of the waveform, or one cycle, is the time interval that repeats. If we find the average of one complete cycle of the waveform, then the average of the next cycle will be exactly the same as the first. If we combine the samples from two cycles, add them together and divide by the total number of samples, we find that not only is the average value of one cycle the same as the average of the second, but also that the average value of two cycles is the same as the average of one cycle. In fact, the average value of any whole number of cycles is the same as the average value of just one cycle. So we can easily find the average value of a very long-duration periodic waveform simply by calculating the average value of one complete cycle.

Equivalent Power

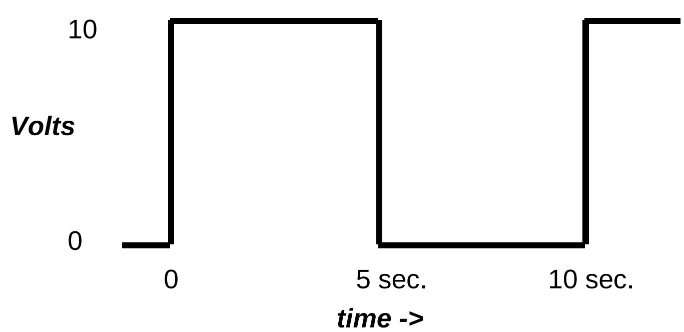
The next important concept is that of equivalent, or heating power. Let’s suppose we put 5 watts into a resistor for 10 seconds. The total amount of energy applied to the resistor is 5 watts X 10 seconds = 50 watt-seconds = 50 joules. This of course raises its temperature by some amount. The actual amount of temperature rise, in degrees, depends on how massive the resistor is, how fast the energy is applied, and how fast the heat can be moved away by conduction, convection, and radiation. So we won’t try to calculate the exact temperature rise. But the important thing here is that *the total amount of heat (energy) the resistor dissipates is the amount of energy that’s applied to it.* Put another way, imagine the resistor being immersed in a thermally insulated tub of water. If we apply 50 joules to the resistor, the water temperature will rise by an amount we can calculate, and it won't matter how we apply that 50 joules – suddenly or slowly – that same temperature rise will result whenever and however we apply 50 joules.

Consider the following simple circuit, where the switch closes and opens, closes

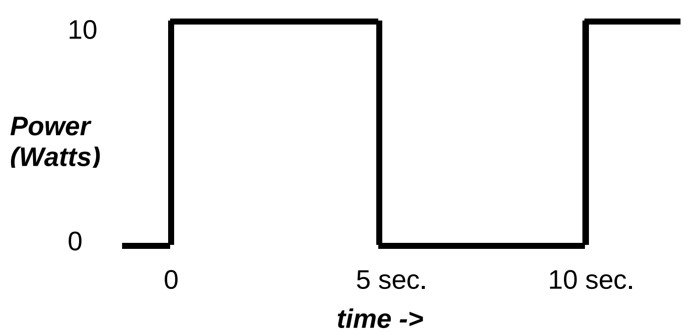
and opens, continuously, with a ten second period:



The voltage applied to the resistor will look like this:



And the power dissipated by the resistor will look like this:



This second waveform of power versus time shows the instantaneous power, that is, the power delivered to the resistor at each instant of time. Too often, it's overlooked that whenever the voltage and/or current varies with time, the power does also. But it's obvious that when the switch is open, no power is being delivered to the resistor. This is extremely important to realize, because it enables us to find the actual amount of energy that's being transferred as well as calculate other important quantities.

As mentioned above, the amount of energy delivered to the resistor is the product of the power and the time. This is strictly true only if the power is constant during that time. When the power varies, the energy can be calculated by sampling the power at frequent time intervals, calculating the power X time product during each interval, and adding them up. The total ends up being the area under the power waveform curve, and it gets more and more accurate as the time interval becomes shorter and shorter. Again, mathematical integration is necessary for all but the simplest curves if extreme accuracy is required. But for our simple power waveform, it's easy to see that in the first five seconds, the area under the curve is simply 10 watts X 5 seconds = 50 watt-seconds = 50 joules. This is the total energy delivered to the resistor during the first five seconds of the period. During the second five seconds, the power is zero, so the energy delivered is zero. So the total energy delivered over one period of ten seconds is 50 joules.

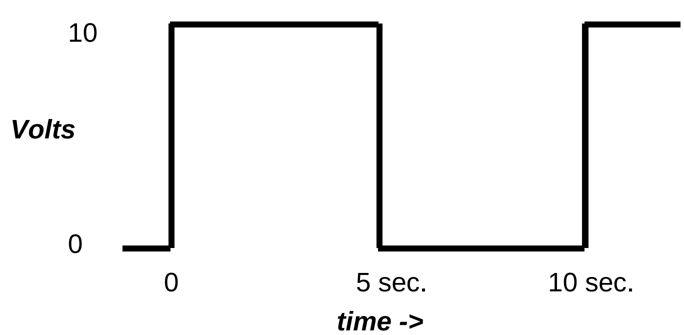
Now let's find the average power during the ten second period.

I think you can see by inspection of the second (power) waveform that the average power is 5 watts. It's 10 watts for half the time and zero for the other half, so if we were to sample it at regular intervals, half the samples would be 10 watts and the other half would be zero. Dividing by the total number of samples, we'd get 5 watts for the average.

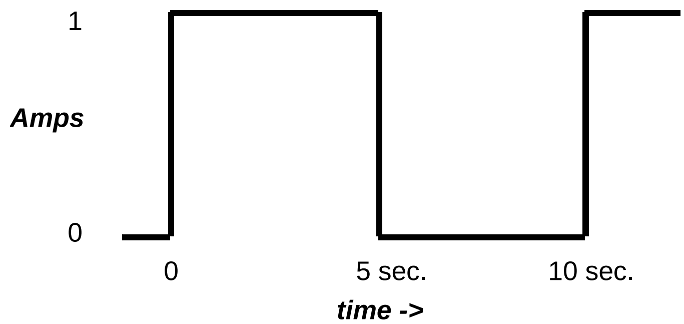
If we had applied 5 watts continuously for the whole 10 seconds of the period, we'd have delivered 50 joules of energy to the resistor. The resistor would dissipate a total of 50 joules, exactly the same amount as it does with the 10 watts of power being delivered for half the time. ***The effective or heating power of our circuit is 5 watts, which is the average power.***

It's easy to see that if we delivered four times the power for ¼ of the time, we'd still have the same average power, and the same total delivered energy – and therefore the same amount of resistor dissipation.

Now let's look at the voltage and current more closely. Recall the voltage waveform:



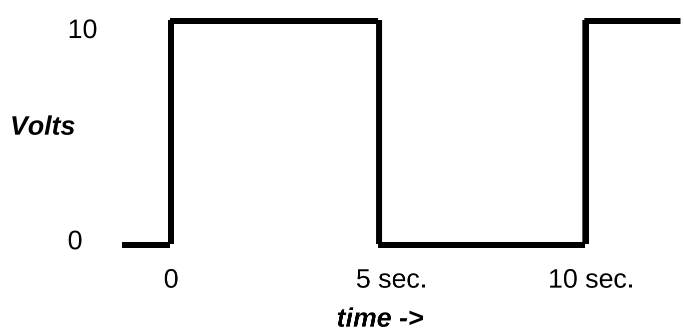
Ohm’s law tells us that the current will be:



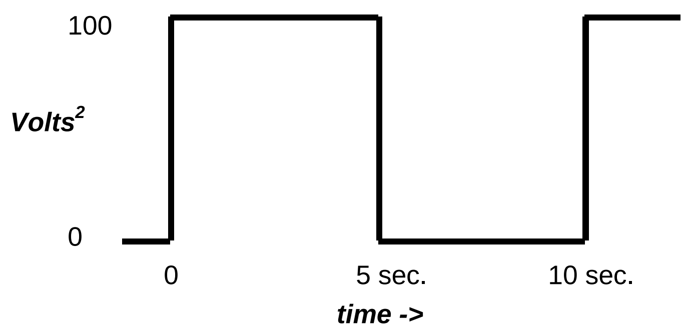
By inspection, the average voltage is 5 volts, and the average current is 0.5 amps. But the average voltage times the average current (= 2.5 watts) is ***not*** the average power. In fact, that product is meaningless. We need some other measure of voltage and current if we’re going to calculate average power without first finding the instantaneous power.

The Meaning of RMS

RMS is a mathematical function, like average, that reduces a complex function to a single value. And, like average, it has a precise definition. The definition is revealed by the name – it’s the square ***R***oot of the ***M***ean of the ***S***quare of the function. Mean is the same as average, so we calculate the RMS value of a function or waveform by first squaring it, then taking the average of the squared function or waveform, then taking the square root of the average. But it has to be done in the proper order! Let’s calculate the RMS value of the voltage waveform from the example. Recalling that the voltage was



The first step is to square the waveform:

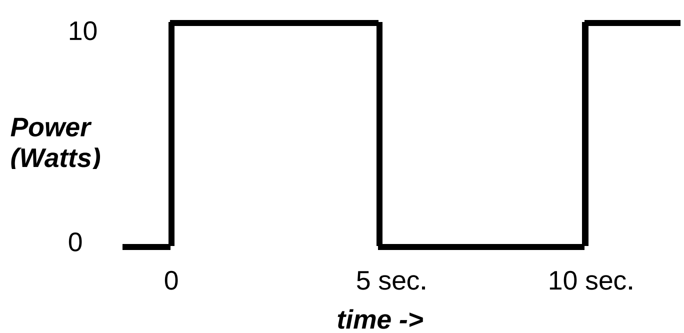


Then we take the average of this waveform, which we can see by inspection is 50 volts^2. The average is no longer a waveform – that is, it doesn't vary with time – it's just a single value. Finally, we take the square root of the average, and get 7.071. . . volts. That's the RMS value of the original voltage waveform. Doing the same for the current waveform, we calculate an RMS value of 0.7071. . . amps. It turns out that when we multiply these together, we get 5 watts – the ***average*** power. This is always true, for any voltage and current waveforms (assuming a resistive load for simplicity). The reason comes from the basic mathematical definitions of RMS and average. It's easiest to see by looking just at the voltage or current. The instantaneous power is $\frac{v^2}{R}$ where v is the instantaneous voltage. Therefore, the average power is $\mathrm{Avg}(\frac{v^2}{R}) = \frac{\mathrm{Avg}(v^2)}{R}$. (R can be moved out of the average since it doesn't change with time.) The RMS voltage V_{RMS} is $\sqrt{\mathrm{Avg}(v^2)}$, so $(V_{RMS})^2 = \mathrm{Avg}(v^2)$, and the average power is $\frac{{V_{RMS}}^2}{R}$. A similar analysis of the current shows that the average power is also $I^2_{RMS}R$. ***The importance of RMS voltage and current are that they can be directly used to calculate the average power.***

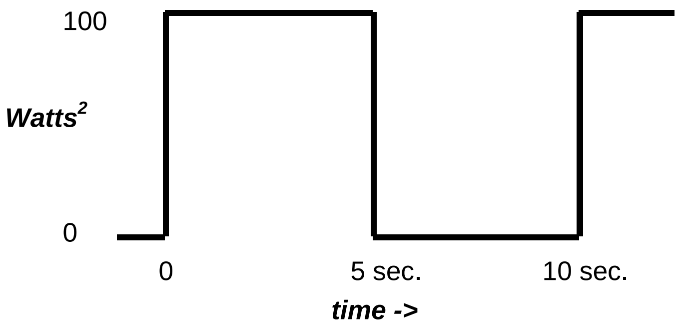
<u>RMS Power</u>

The RMS value of a power waveform can be calculated just like the RMS value of any other waveform, although it doesn't represent heating power or any other useful quantity. I'll do it just to show how it would be done.

Recall that the power was:



Squaring it, we get



The average of the square of the power is clearly 50 watts, and the square root of that is 7.0711. . . watts. We found earlier that the equivalent heating power of our circuit – the **average** power -- was 5 watts, not 7. ***The RMS value of power is not the equivalent heating power and, in fact, it doesn't represent any useful physical quantity.*** The RMS and average values of nearly all waveforms are different. A notable exception is a steady DC waveform (of constant value), for which the average, RMS, and peak values are all the same.

It should be noted that the term "RMS power" is (mis)used in the consumer audio industry. In that context, it means the average power when reproducing a single tone, but it's not actually the RMS value of the power.

Summary

I've shown that:

- The equivalent heating power of a waveform is the average power.
- The RMS power is different than the average power, and therefore isn't the equivalent heating power. In fact, the RMS value of the power doesn't represent anything useful.
- The RMS values of voltage and current are useful because they can be used to calculate the average power.

For those who are interested, mathematical derivations are readily available and can be found on various web sites and in textbooks. What I've tried to do here is to present the basic concepts with as little mathematics as possible. Questions are welcome – send them to *w7el@eznec.com*. And I'd appreciate very much if anyone finding an error would notify me so it can be corrected.

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