

The “Food for thought” series was written to clarify some issues raised in an ongoing discussion on the rec.radio.amateur.antenna newsgroup regarding transmission lines. They’re an attempt to explain a number of frequently misunderstood phenomena, and put to rest some myths. Those who don’t want to deal with the moderate amount of math should still be able to get quite a bit from the discussion and conclusions.

They were originally posted from August 17 to August 21, 2002 on the thread titled “Superposition quiz”. The reader is encouraged to do a Google newsgroup search and read some of the questions and comments generated by the postings.

This folder contains the individual postings, plus a text file (Food for thought.txt) containing all the postings in the order they were written. Each one is reasonably self contained, although occasional references are made to previous ones. In order, they are:

- Superposition
- Source reflections
- Instantaneous and average power
- Waves and power
- Forward and reverse power

I encourage people to share and reproduce these, and ask only that the source be credited.

Reports of any errors are welcome.

Roy Lewallen, W7EL

Food for thought: Superposition

Superposition is a simple concept, but like other simple concepts, it can be and often is misapplied.

The test for superposition is as follows:

If excitation $x_1(t)$ produces response $f(x_1(t))$ and $x_2(t)$ produces response $f(x_2(t))$, then $x_1(t) + x_2(t)$ produce $f(x_1(t) + x_2(t))$.

Let’s take a simple example. Connect a 50 volt DC voltage source (V_1) in series with a 30 volt DC voltage source (V_2) (+ to -), and this in series with a 20 ohm resistor @ to make a simple loop. We know that $I = V/R$, so we can say:

$$\begin{aligned} I &= f(x) = x/20 \text{ ohms} \\ V_1 &= x_1 = 50 \text{ volts} \\ V_2 &= x_2 = 30 \text{ volts} \end{aligned}$$

Let’s see if the voltages and currents can be superposed. Mathematically,

$$\begin{aligned} I_1 &= f(50 \text{ volts}) = 50 \text{ volts}/20 \text{ ohms} = 2.5 \text{ amperes} \\ I_2 &= f(30 \text{ volts}) = 30 \text{ volts}/20 \text{ ohms} = 1.5 \text{ amperes} \\ f(50 + 30 \text{ volts}) &= (50 \text{ volts} + 30 \text{ volts}) / 20 \text{ ohms} = 4 \text{ amperes} = 2.5 + 1.5 \end{aligned}$$

So the voltages satisfy the requirement. In our circuit, we can do this test by turning off all the sources (replacing them with short circuits, since the impedance of a voltage source is zero), then turning them on one at a time. With only V_1 on, $I = I_1 = 50/20 = 2.5$ amps. With only V_2 on, $I = I_2 = 30/20 = 1.5$ amps. With both on, $I = (50 + 30)/20 = 80/20 = 4$ amps. This is the sum of the currents we got from each source by itself.

We can turn the 30 volt source around the other way so it subtracts from the 50 volt source, and still have superposition satisfied. In that case,

$$\begin{aligned} I_1 &= 50/20 = 2.5 \text{ amperes} \\ I_2 &= -30/20 = -1.5 \text{ amperes} \\ I_1 + I_2 &= (50 - 30) / 20 = 1 \text{ ampere} \end{aligned}$$

The conditions for superposition are pretty strict. For example, we can't have a source that's one value when we turn it on one time and another value when we turn it on another time. And we can't have a voltage or current source whose value changes depending on the load. But we can have time varying sources like sine waves, providing the voltage-time function they produce stays constant. And we can have, say, a source with 50 ohm impedance. Just replace a source with a box containing a voltage source in series with a 50 ohm resistor. Turn the source off and on as before, and you'll see that superposition is still satisfied—all we've done is add a resistor to the circuit. And even a simple function like the familiar first order linear equation $f(x) = mx + b$ doesn't satisfy superposition.

Let's see if the powers produced by the sources can be superposed. Turning the 30 volt source back around so its voltage adds to the 50 volt source, we define the function relating power to voltage.

$$P = f(x) = x^2/20 \text{ ohms}$$

$$V1 = x1(t) = 50 \text{ volts}$$

$$V2 = x2(t) = 30 \text{ volts}$$

Test for superposition:

$$P1 = f(50 \text{ volts}) = 50^2/20 = 125 \text{ watts}$$

$$P2 = f(30 \text{ volts}) = 30^2/20 = 45 \text{ watts}$$

$$f(50 + 30 \text{ volts}) = (50 + 30)^2/20 = 320 \text{ watts.}$$

$f(50 + 30)$ isn't equal to $f(50) + f(30)$, so this function (power) doesn't satisfy superposition. We can see this with the circuit by turning each source on in turn and observing the power dissipated by the resistor, then turning both on at the same time and looking at the power.

The test for superposition is simple, so before declaring that superposition holds for a particular quantity or circuit, first plug the values into the test and see if what you say is true.

Roy Lewallen, W7EL

- Thanks to Cecil Moore, W5DXP for bringing to my attention a minor error which has now been corrected. It does not impact the conclusions.

Food for thought: Source reflections

Here's an insight that might help you understand forward and reflected waves a little better.

Consider a transmission line at the moment a signal is first applied. A wave will travel down the line, with voltage and current related by $Vf/If = Z0$. The wave hits the end of the line, and some fraction of it will be reflected. This fraction, which can also include a phase shift, is the reflection coefficient GAMMA at the end of the line. (Lower case rho is also sometimes used for the complex reflection coefficient, but it's also sometimes used for the magnitude only.) The reflection coefficient at the end of the line is determined solely by the load impedance Zl and the line's characteristic impedance Z0: $GAMMA = (Zl-Z0)/(Zl+Z0)$. It's a simple relationship, but you have to remember that Zl can be complex and, if the line has loss, Z0 can also be complex.

If Zl isn't equal to Z0, GAMMA won't be zero, and GAMMA times the forward wave is reflected at the end of the line and travels back toward the source as a reflected wave. When it reaches the input end of the line, we've established standing waves along the line due to the interference between the forward and reverse waves. The SWR is, in fact, $(1 + rho)/(1 - rho)$, where rho is the magnitude of the reflection coefficient GAMMA. We can also calculate an impedance V/I at any place along the line.

If the source impedance isn't the same as the line impedance, some fraction of the reverse travelling wave will be re-reflected toward the source. And now we come to the important point that I believe people often don't think about. Whatever fraction of the reverse wave is re-reflected back toward the load, GAMMA times that amount will be reflected back toward the source when it reaches the load end. So this re-reflection adds some amount (depending on the source mismatch) to the initial forward wave, and GAMMA times that amount to the reverse travelling wave. It changes both the forward and reverse travelling waves in the same proportion as the original pair of waves! So THE RATIO OF THE FORWARD AND REVERSE TRAVELLING WAVES ISN'T CHANGED BY THE REFLECTION

FROM THE SOURCE. The amplitude and perhaps phase of the forward and reverse travelling waves change as re-reflections continue to occur until steady state is reached. BUT THE RATIO OF THE FORWARD AND REVERSE TRAVELLING WAVES STAYS THE SAME AS IT WAS BEFORE ANY RE-REFLECTIONS TOOK PLACE FROM THE SOURCE. THE SOURCE MISMATCH HAS NO EFFECT ON THE RATIO OF FORWARD AND REVERSE WAVES. Therefore, the source mismatch has nothing to do with the SWR. This is something that hopefully most people know, but too many people attribute too much importance to the source match.

If we change the source match, the magnitude and phase of the source reflections change, altering the magnitude and phase of the forward and reverse travelling waves. In other words, we're adjusting the amount of voltage and current (and therefore power) we're putting into the line. But we're not changing the SWR or the ratio of the forward and reverse travelling waves. That's determined solely by the load impedance and line Z_0 .

Roy Lewallen, W7EL

Food for thought: Instantaneous and average power

Power is the rate of doing work, or expending or transferring energy. Because these quantities can vary with time, power can also be a function of time. If power is used to express movement of energy (such as the power in a radiated field), a direction can also be assigned to it.

In terms of voltage and current, power is simply calculated as:

$$P(t) = v(t) * i(t)$$

where $v(t)$ and $i(t)$ can be any arbitrary functions of time.

We're often interested in the steady state case, where $v(t)$ and $i(t)$ are sine waves of the same frequency. In that case, we can represent $v(t)$ and $i(t)$ as:

$$v(t) = V * \cos(\omega * t + \phi_{iv})$$

$$i(t) = I * \cos(\omega * t + \phi_{ii})$$

where

V = the peak (not RMS) value of the voltage

I = the peak (not RMS) value of the current

$\omega = 2 * \pi * \text{frequency}$

t = time

ϕ_{iv} = phase angle of the voltage

ϕ_{ii} = phase angle of the current

Multiplying to get power,

$$P = V * I * \cos(\omega * t + \phi_{iv}) * \cos(\omega * t + \phi_{ii})$$

Using the trig identity

$$\cos(a) * \cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

the power equation can be rewritten

$$P = (V * I)/2 * [\cos(\phi_{iv} - \phi_{ii}) + \cos(2 * \omega * t + \phi_{ii} + \phi_{iv})]$$

It's evident that this is the sum of two quantities,

$$(V * I)/2 * \cos(\phi_{iv} - \phi_{ii}), \text{ and}$$

$$(V * I)/2 * \cos(2 * \omega * t + \phi_{ii} + \phi_{iv})$$

The first term is a constant value that doesn't vary with time. The second is a sine wave, with frequency equal to twice the frequency of v or i . So the power does change with time. If we graph it, we'll see a sine wave (the second term above) with frequency of twice that of v or i , with an offset equal to the first term above.

Let's look at some interesting cases. If v and i are in phase, $\phi_{iv} - \phi_{ii} = 0$, and the offset (the first term above) will be equal to $(V * I) / 2$. This will bring the entire waveform above the horizontal axis, so the power will never be negative. The power will touch zero just for an instant at the most negative part of each cycle.

If v and i are 90 degrees out of phase (it doesn't matter whether the difference is + or - 90 degrees, since $\cos(-x) = \cos(x)$), the first term will be zero. The power waveform will then be a sine wave centered around zero, and will be positive for half of each cycle, and negative for the other half. I'll talk about what this means in a moment.

If v and i are between 0 and +/-90 degrees, the first term will be positive, but not enough to bring the whole power waveform above zero. So it will be positive for most of each cycle, but negative for part.

v and i will be in phase if the load is purely resistive. That is, the impedance Z relating v and i ($Z = v/i$) is purely real. In that case, power always flows toward the load (actually, in the direction defined as positive for I), although its amplitude varies during the cycle.

If Z is a pure reactance like an inductance or capacitance would present, then ϕ_{iv} and ϕ_{ii} would be 90 degrees apart. There's still energy flowing in the circuit, but it flows one way for half the cycle and the other way for the other half. In the case of a purely capacitive load, the energy is accumulated and stored in an electric field for half the cycle, then returned during the other half. The same thing happens with a purely inductive load, except the energy is stored in a magnetic field. It's very important to realize that power is moving back and forth in this circuit, even though it's not doing any work. Because it doesn't do work, the power in this situation is sometimes called "imaginary", "reactive", or "VAR" (volt-amp-reactive) power. But it can be calculated, detected, and measured flowing in the circuit just as surely as power that does work.

So how do we find out how much work is being done? The rigorous way is to integrate the power over the period of interest. This works for any power waveform, not just the sinusoidal steady state waveform. But a way that works as well, and amounts to the same thing, is to calculate the average power, which will be a constant value for the steady state condition if calculated over an integral number of cycles. This is multiplied by the time to get the work done or energy expended, or the net energy moved. There's a standard way to calculate the average value of any periodic waveform, and that could be done here. But for the steady state condition, a simple inspection of the two terms making up the power waveform shows that the first term IS the average value. The average of the second term is zero, so it contributes nothing to the overall average. So the average power is simply

$$(V * I)/2 * \cos(\phi_{iv} - \phi_{ii})$$

which is equal to

$$V_{rms} * I_{rms} * \cos(\phi_{iv} - \phi_{ii})$$

Notice that if the load is purely reactive, the average is zero and no work is being done. No work is done by energy that leaves the source then is returned.

Although the average power is useful in determining how much work is done, energy expended, or net energy moved, remember that it doesn't tell the whole story. Keep in mind the statistician who drowned while crossing a creek whose average depth was only three feet.

One final caution: Be careful when using phasor quantities to calculate power. Phasors are not exact representations of the corresponding time waveforms, and $\sim V * \sim I$, where $\sim V$ and $\sim I$ are phasors, is not power or any other useful quantity. Among other problems, phasors have a built in assumption that all quantities have the same frequency or are D.C. So for that reason alone there's no way to express power as a single phasor quantity or single quantity plus a D.C. term as a function of V and/or I , since V and I have different frequencies than P . It does turn out that $\text{Re}\{\sim V * \sim I(\text{conj})\}$, where $\sim I(\text{conj})$ is the complex conjugate of $\sim I$, or $\text{Re}\{\sim V(\text{conj}) * \sim I\}$ does happen to be the average power. So it's possible to calculate the average power from phasors, but not to express power as a phasor quantity in the same equation with V or I .

Roy Lewallen, W7EL

Food for thought: Waves and power

In a transmission line under steady state conditions, we can conveniently separate the voltage and current into forward and reverse travelling voltage and current waves. In actuality, electric and magnetic fields were created when power was first applied to the line, and if both source and load are mismatched, an infinite number of reflections took place from both ends of the line. And arguments can be made that fields themselves are mathematical constructs. But the simplified model of single forward travelling V and I waves and single reverse travelling V and I waves are adequate to explain virtually everything about voltage, current, impedance, and power that can be observed and measured. Hence, the model is well accepted and in general use.

The following discussion will be restricted to steady state analysis with sinusoidal voltages and currents.

The forward and reverse travelling voltage and current waves add at each point along the line. At any point l along the line,

$$\begin{aligned}v_l &= f_v l + r_v l \\i_l &= f_i l + r_i l\end{aligned}$$

where f and r denote forward and reverse waves respectively, and v_l , $f_v l$, $r_v l$, i_l , $f_i l$, and $r_i l$ are all functions of time (sinusoids). Note that when using the above formula for i_l , the positive direction of current is physically the same (toward the load) for both the forward and reverse waves. With this convention, the sign of i changes when it reflects from the end of the line.

At any point along the line, we can calculate a power. We can calculate the power directly from the time functions of voltage and current. In a lossless line with resistive load, this isn't difficult, but it becomes pretty ugly in the general case where the line has loss, and reflection coefficient and line characteristic impedance can be complex. In the general case, it's not too difficult to calculate average power from phasor representations of v and i .

The power at the input of the line, or any point l along the line is:

$$p_l = v_l * i_l$$

where all three quantities are functions of time.

“Forward” and “reverse” powers are often calculated as follows:

$$\begin{aligned}f_p &= f_v * f_i \\r_p &= r_v * -r_i\end{aligned}$$

The negative sign for r_i is because of the common convention that the positive direction for the reverse power is in the direction of flow, i.e., from the load toward the source, while the convention followed for r_i is from the source to the load.

Because $p = v * i$, and v and i are each the sum of a forward and reverse component, we can write that at any point l ,

$$p_l = (f_v l + r_v l) * (f_i l + r_i l)$$

and, expanding,

$$p_l = f_v l * f_i l + r_v l * r_i l + f_v l * r_i l + r_v l * f_i l$$

A popular concept is to consider the “forward” and “reverse” powers as being waves travelling along with the voltage and current, and the net power being moved at any point being equal to their difference, that is,

$$p_l = f_p l - r_p l$$

Substituting from equations above,

$$p_l = f_v l * f_i l - (r_v l * -r_i l) = f_v l * f_i l + r_v l * r_i l.$$

Unfortunately, this isn't the same result as we got by directly calculating the power from the total voltage and current at a given point—two terms are missing. The product of the sum of the waves isn't equal to the sum of the products. The net power isn't the same as the difference between the forward and reverse powers. (Don't blow your stack yet. I said they're not the same, not that they can't be equal or nearly equal. Read on.)

If we measure i and v at the input end of the line and multiply them, the result is the power entering the line. The average value of that power is the net amount of power being delivered to the input end of the line that can do work. (From an earlier discussion, the time power waveform can include power (so-called reactive power) being pushed into the line and being returned each cycle. This power doesn't do work, and doesn't show up as part of the average.) So the actual power being delivered to the line is the power calculated by first summing the forward and reverse voltage and current waves, not the difference between the forward and reverse power. Because we can take the average of each additive term in either expression and add them to get the overall average, the same problem exists for the averages as for the time waveforms themselves.

But directional wattmeters work and give useful results. Why?

The reason is that AS LONG AS THE LINE Z_0 IS ENTIRELY REAL (RESISTIVE), THE CROSS TERMS $v_1 * i_1$ AND $v_1 * i_1$ ARE EXACTLY THE NEGATIVE OF EACH OTHER AND CANCEL. This happy result occurs because of the unique relationship between the forward and reverse voltage and current waves, and between current and voltage. As long as Z_0 is purely real, we can calculate the actual power flow by subtracting the reverse power from the forward power.

What happens when Z_0 is complex? The cross terms are then functions of not only the phase of Z_0 , but also the magnitude and phase of the reflection coefficient, and the length and attenuation of the cable. So the difference between the actual power and the power calculated by subtracting "reverse" from "forward" power depends on all these quantities.

As it turns out, Z_0 can be considered to be purely real in the vast majority of amateur antenna situations. Reactive Z_0 is caused by high loss per wavelength. As frequency increases, wavelength decreases in inverse proportion to the frequency, but loss per unit length increases only as the square root of frequency. (This is true as long as conductor loss dominates, which is the case for coax cables with decent dielectric up to 1 GHz or higher.) So for a given cable, the loss per wavelength decreases as frequency increases. Consequently, reactive Z_0 occurs at low, not high, frequencies, and is more pronounced with lossier cables. At 3.5 MHz, the impedance of RG-58 is about $50.0 - j1.2$ ohms. At 1.8 MHz, it's $50.0 - 1.7$. It's not likely that this amount of reactance would cause objectionable errors in power calculations. About the most extreme more-or-less practical case would be RG-174 at 1.8 MHz, which has a Z_0 of about $50.4 - j5.9$ ohms. That might produce a noticeable error when calculating power from forward and reverse, but this isn't a common combination of cable and frequency.

Lowfers should take note, though. Below the AM broadcast band, some cables could have a significantly reactive Z_0 .

Directional wattmeters such as the Bird indirectly calculate the power in an internal line which presumably has a very nearly pure real Z_0 , so they should remain accurate even if connected lines have complex Z_0 .

One of my college professors made a statement that stuck with me: "Progress hasn't been set back so much by getting the wrong results with the right methods as it has by getting the right results by the wrong methods." When you subtract reverse power from forward power, be aware that you're not actually calculating the total power. You're just getting close enough for practical purposes. Usually.

Roy Lewallen, W7EL

Food for thought: Forward and reverse power

This essay discusses transmitter models, forward and reverse power, what happens to reverse power when it encounters the source impedance, "mismatch loss", and recovering reverse power with a circulator.

In the following discussion I'm going to assume for simplicity that transmission line loss is zero.

When this is true, the average power furnished by the source (which I'll call the net average power) equals the average power delivered to the load. And previous analysis revealed that when Z_0 is purely real, as it is when the line is lossless, we can separate the average power into "forward" and "reverse" components whose difference equals the net average power furnished by the source or delivered to the load. The average forward power can be computed from the forward travelling voltage and current waves, and the average reverse power can be computed from the reverse travelling voltage and current waves.

My commercial amateur HF transceiver is probably typical of modern rigs in that it produces a constant **forward** average power into varying load impedances—provided the impedance isn't extreme enough to cause the rig to severely cut back its power output. I could combine the transmitter with a transmatch to make a system that would deliver a constant **net** average power into any load within the adjustment range of the matching network. Of course, the matching network would have to be adjusted each time the load impedance is changed.

It turns out that a linear model of my transmitter (without a transmatch) over its non-shutdown range is very simple—it's just a voltage source in series with a resistance. This combination produces a constant average forward power into a transmission line connected to any load impedance, if the series resistance equals the characteristic impedance of the line. I'm going to present this without proof, although the calculation is straightforward from fundamental transmission line equations. Any example you can devise will confirm it holds true.

I make no claim that the model circuit represents what's going on **inside** the transmitter. For one thing, a real transmitter will typically be more efficient than the model. However, if measurements and observations are limited to the outside of the transmitter, the model is very good (within the non-shutdown range). Although a real transmitter won't contain the model's resistance as a resistor, we will take a look at the model resistor's dissipation to see how it interacts with the "reverse" power.

The average forward power fPa fed into a line with (purely real) characteristic impedance $R0$ by a voltage source of RMS voltage $Vrms$ in series with a resistance $R0$ is

$$fPa = Vrms^2 / (4 * R0)$$

So we can model a 100 watt forward, 50 ohm nominal transmitter as a 141.4 volt ($100 * \sqrt{2}$) RMS voltage source in series with a 50 ohm resistance. If we connect this to a 50 ohm transmission line of any length, we can put on any load impedance we like, INCLUDING A SHORT OR OPEN CIRCUIT, and we'll have 100 watts of average forward power in the 50 ohm line. In terms of the impedance $Zi = Ri + jXi$ looking into the line, and the source resistor and line's (assumed purely real) characteristic impedance of $R0$,

$$fPa = Vrms^2 / (4 * R0)$$

$$rPa = \{ Vrms^2 / (4 * R0) \} * \{ [(Ri - R0)^2 + Xi^2] / [(Ri + R0)^2 + Xi^2] \}$$

where fPa and rPa are the average forward and reverse power, respectively. The net average power from the source and into the load $Pa = fPa - rPa$. Another useful relationship, although it won't be used here, is that $rPa/fPa = \rho^2$, where ρ is the magnitude of the reflection coefficient at the load end of the $R0$ line.

Let's take a look at a few simple cases. Connect the "transmitter" through a half wavelength 50 ohm line to a load. (I'm choosing a half wavelength line just to make the input impedance easy to calculate. The forward and reverse powers are independent of the line length if the line is lossless as it is here.) So $Zi = Zl$, the load impedance on the far end of the line. Also notice that the forward power doesn't depend at all on Zi , hence Zl , so it will be 100 watts for all the following examples.

If $Zl = 50 + j0$, a matched load, then from the equation above, $rPa = 0$. If $Zl = 100 + j0$, one of the infinite number of possible loads causing a 2:1 SWR on the line, $rPa = 11$. If $Zl = 25 + j0$, another 2:1 SWR condition, $rPa = 11$. $Zl = 37 + j28$ is yet another (nearly) 2:1 load, and $rPa = 11$ as it should. It's easy to see that if the load is a pure reactance ($Rl = 0$), $rPa = fPa = 100$ watts, and if $Rl = 0$, $rPa = 100$ watts. If the load end of the cable is open circuited, $Rl = Rl$ gets very large. Because it's much larger than $R0$ or Xi , the second bracketed quantity in the rPa equation approaches 1, so again $rPa = fPa = 100$ watts.

Although I've said that the "transmitter" model doesn't represent the inside of a transmitter, it does represent the inside of a signal generator, which typically does have a physical source resistor. It also represents, well, a voltage source in series with a resistor. Let's take a look at the resistor's dissipation.

Here, we can do a very simple analysis. As seen from the source resistance, the load Zi is simply Zl , since there's no impedance transformation by the half wavelength of transmission line. So the circuit is a simple voltage divider. From this we can do the following calculations. I = the current from the source, through the source resistor and through the load. (The magnitude of the current isn't transformed by the half wavelength line either.) With source resistor being $R0$, $I_{rms} = Vrms / (R0 + Zl)$. Power dissipated by $R0 =$

$$Pa(R0) = |I_{rms}|^2 * R0 = (Vrms^2 * R0) / [(R0 + Rl)^2 + Xi^2]$$

and the power dissipated by the load =

$$Pa(Rl) = |I_{rms}|^2 * Rl = (V_{rms}^2 * Rl) / [(R0 + Rl)^2 + Xl^2]$$

Note that the net average power dissipated by the load $Pa(R0)$ equals $Pa(tx)$, the net average power produced by the “transmitter”, which equals $fPa - rPa$. The net average power produced by the “transmitter’s” voltage source $Pa(src)$ is this amount plus the amount dissipated in the “transmitter’s” source resistance, or $Pa(R0) + Pa(Rl)$. The fraction of the total source power dissipated by the load is, from the above equations, $frac Rl = Rl / (R0 + Rl)$. The fraction of the total power dissipated by the source resistor is $frac R0 = R0 / (R0 + Rl)$.

From this, we can calculate:

Zl	fPa	rPa	Pa(tx)	Pa(src)	Pa(R0)	Pa(Rl)	frac R0	frac Rl
50 + j0	100	0	100	200	100	100	0.50	0.50
100 + j0	100	11.1	88.9	133	44.4	88.9	0.33	0.67
25 + j0	100	11.1	88.9	267	178	88.9	0.67	0.33
37 +/-j28(*)	100	11.4	88.6	209	120	88.6	0.58	0.42
0 +/-j50	100	100	0	200	200	0	1.00	0
0 +/-j100	100	100	0	80.0	80.0	0	1.00	0
0 + j0	100	100	0	400	400	0	1.00	0
infinite	100	100	0	0	0	0	-	-

(*) For any Zl that causes exactly a 2:1 SWR, rPa will equal 11.1 and $Pa(Rl) = 88.9$. The values shown for 37 +/-j28 are slightly different because this impedance doesn’t result in quite exactly a 2:1 SWR.

For the second, third, and fourth entries, the SWR is 2:1. The forward and reverse powers are the same for all three, and the source impedance (50 ohms) is the same for all the above cases. So here we have three cases where the reverse powers are the same, and the impedance match looking back toward the source is the same (1:1), yet the dissipation in the source resistor $Pa(R0)$ is very different. The obvious conclusion is that **THE POWER DISSIPATED IN THE SOURCE RESISTANCE ISN’T DETERMINED DIRECTLY BY THE SOURCE MATCH, THE SWR, OR THE REVERSE POWER**. Otherwise it would be the same in all three cases, since all these quantities are the same for all three.

For the last four entries, the SWR is infinite, and the reverse power is a full 100 watts. The source is perfectly matched to the line for all table entries. Yet the source resistor dissipation varies from 0 to 400 watts depending on the load impedance – despite no difference in source match, or forward or reverse power for the four entries.

The last two entries are particularly interesting. When the line is open circuited at the far end (last table entry), there is no power at all dissipated in the source resistor. So **none** of the reverse power is dissipated in the source resistor. Yet when the line is short circuited at the far end (next to last table entry), the source resistor dissipates *twice the sum of the forward and reverse powers*.

From the last entry alone we can conclude that **THE REVERSE POWER IS NOT DISSIPATED IN OR ABSORBED BY THE SOURCE RESISTANCE**. And the table clearly shows that the source resistor dissipation bears no relationship to the amount of reverse power.

I personally don’t have a compulsion to understand where this power “goes”. The power is actually a function of time, and here we’re only looking at the average power. We’ve calculated “forward” and “reverse” power from the forward and reverse travelling voltage and current waves. The “reality” of these moving waves of time varying or average power can be argued either way. While the nature of the voltage and current waves when encountering an impedance discontinuity is well understood, we’re lacking a model of what happens to this “reverse power” we’ve calculated. And ascribing to it the same characteristics of reflection as for voltage and current invariably lead to demonstrably wrong conclusions. **ANY MODEL PRESENTED TO ACCOUNT FOR WHAT HAPPENS TO “FORWARD” AND “REVERSE” POWER AT TRANSMISSION LINE ENDS HAD BETTER GIVE RESULTS THAT AGREE WITH THE ABOVE TABLE**. Otherwise, it’s wrong. The values in the above table can be measured and confirmed.

A final observation about this analysis is that the average power dissipated by the *load* resistance is the same for all three cases where the SWR is 2:1 (see note *). In fact, it will be this value for any impedance resulting in a 2:1 SWR on the cable (reflection coefficient magnitude of 1/3). The ratio of the load power when the load is matched (100 watts) and when mismatched (88.9 watts for 2:1 SWR) is termed the “mismatch loss”. It has nothing to do with cable loss—in the above example, the transmission line has no loss at all, yet the mismatch loss for 2:1 SWR is about 0.5 dB. There’s no part of the circuit we can point to which is dissipating the “missing” 11.1 watts, because “mismatch loss” doesn’t

represent dissipation anywhere. It's simply a measure of the amount of power we could have had if we had matched the load versus what we get with a mismatched load. If we use a transmatch at the source or load, we can obtain the full 100 watts in the same load resistor from the same "transmitter", reducing the "mismatch loss" to zero. (Or we can consider the transmatch to have "mismatch gain" compensating for the "mismatch loss".) Mismatch loss is a useful concept when dealing with fixed impedance sources, but not when the impedance is adjustable. And it should never be confused with actual dissipation anywhere in the circuit, as it often is.

Some inventive people have supposed they can separate forward and reverse power with a circulator. That really sounds attractive, particularly with an open or short circuited load. In that condition, the forward and reverse powers are each 100 watts, yet the transmitter (or the transmitter voltage source) doesn't have to produce any power at all. If we could separate the forward and reverse powers and deliver them to loads, we'd have 200 watts of free power. Even just one of them would give us 100 watts to sell back to the power company. Expect to find something about this in some future April first posting. Seriously, though, if we put a circulator in the line, the source would see a perfect match, so the transmitter would produce a net average power of 100 watts, 100 forward and zero reverse. Past the circulator, there would be 100 watts forward and 100 reverse like before (presuming an open or short circuit at the load end of the line). 100 watts would exit the circulator's "reverse" port. I'll leave it to the philosophers to argue whether that 100 watts is the 100 watts produced by the generator or the 100 watts of "reverse" power past the circulator. The fact is that 100 watts went into the line/circulator/load system, and 100 watts came out. No surprise there. But don't expect to get one microwatt more than that out of the system, no matter how clever you are.

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- Thanks to Cecil Moore, W5DXP for bringing to my attention several typos and a point benefiting from clarification which have now been corrected. None impact the analysis or the conclusions.
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